In a nutshell: Laplace's equation

Given a region that is a subset of a rectangle or three-dimensional orthotope (a rectangular parallelepiped) that is defined by $[a_x, b_x] \times [a_y, b_y]$ or $[a_x, b_x] \times [a_y, b_y] \times [a_z, b_z]$ such that there exists an h and integers n_x, n_y and possibly n_z such that $h = \frac{b_x - a_x}{n_x} = \frac{b_y - a_y}{n_y}$ and possibly $h = \frac{b_z - a_z}{n_z}$, suppose we want to approximate the solution to Laplace's equation $u(x, y)$ or $u(x, y, z)$.

1. Set $x_i \leftarrow a_x + ih$ noting that $x_{n_x} = b_x$, $y_j \leftarrow a_y + jh$ noting that $y_{n_y} = b_y$, and possibly $z_k \leftarrow a_z + kh$ noting that $z_{n_z} = b_z$.
2. Create an $(n_x + 1) \times (n_y + 1)$ 2D-array or $(n_x + 1) \times (n_y + 1) \times (n_z + 1)$ 3D-array $\mathbf{u}$ where $u_{i,j}$ will approximate $u(x_i, y_j)$ or where $u_{i,j,k}$ will approximate $u(x_i, y_j, z_k)$.
3. Some points will have fixed values, others will be insulated, and others will be unknown. It is essential that all values along the boundary of the rectangle or 3-dimensional orthotope either have fixed values or be insulated—they cannot be unknown.
4. Let N be the number of unknown values. Associate each unknown value with an unknown ranging from v_1 to v_N. For example, we may have the following where insulated boundary values are marked with *, fixed boundary values are marked with a real number, and there are 22 unknown interior points, each associated with an index into a one-dimensional vector $\mathbf{v}$.

$$\begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ * & u_{1,1} = v_1 & u_{1,2} = v_2 & u_{1,3} = v_3 & u_{1,4} = v_4 & u_{1,5} = v_5 & u_{1,6} = v_6 & 5 \\ * & u_{2,1} = v_7 & u_{2,2} = v_8 & u_{2,3} = v_9 & u_{2,4} = v_{10} & u_{2,5} = v_{11} & u_{2,5} = v_{12} & 5 \\ * & u_{3,1} = v_{13} & u_{3,2} = v_{14} & * & u_{3,4} = v_{15} & u_{3,5} = v_{16} & u_{3,5} = v_{17} & 5 \\ * & u_{4,1} = v_{18} & u_{4,2} = v_{19} & * & u_{4,4} = v_{20} & u_{4,5} = v_{21} & u_{4,5} = v_{22} & 5 \\ 5 & 5 & 5 & 5 & 25 & 25 & 5 & 5 \end{pmatrix}$$

5. For each unknown, write down the following equation: $4u_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1} = 0$ or, if applicable, $6u_{i,j,k} - u_{i-1,j,k} - u_{i+1,j,k} - u_{i,j-1,k} - u_{i,j+1,k} - u_{i,j,k-1} - u_{i,j,k+1} = 0$. Each of these entries is associated with either a boundary value, an insulated boundary, or an unknown v_ℓ. For each fixed value, just substitute it into the equation. For each insulated boundary point, replace it with the unknown v_ℓ associated with the point $u_{i,j}$ or $u_{i,j,k}$, and for each unknown, substitute it with its corresponding unknown v_ℓ.

   For example, we have $4u_{4,2} - u_{3,2} - u_{5,2} - u_{4,1} - u_{4,3} = 0$, so $4v_{19} - v_{14} - v_{19} - v_{18} - 5 = 0$, with one insulated point and one boundary point.
6. Having done this for each unknown value, this defines a system of N linear equations in N unknowns. Solve this system of linear equations. The solution v_ℓ is the approximation of the corresponding value $u(x_i, y_j)$ or $u(x_i, y_j, z_k)$.

   For example, having solved we get that $v_{19} = 5.22$, so $u_{4,2} = 5.22$.

Acknowledgement: Jakob Koblinsky noted the incorrect indices in the Cartesian products and Step 1.

For your viewing pleasure, the above state defines this system of of 22 linear equations in 22 unknowns:

$$\begin{pmatrix}
3 & -1 & & & & & -1 & & & & & & & & & & & & & & & \\
-1 & 4 & -1 & & & & & -1 & & & & & & & & & & & & & & \\
 & -1 & 4 & -1 & & & & & -1 & & & & & & & & & & & & & \\
 & & -1 & 4 & -1 & & & & & -1 & & & & & & & & & & & & \\
 & & & -1 & 4 & -1 & & & & & -1 & & & & & & & & & & & \\
 & & & & -1 & 4 & & & & & & -1 & & & & & & & & & & \\
-1 & & & & & & 3 & -1 & & & & & -1 & & & & & & & & & \\
 & -1 & & & & & -1 & 4 & -1 & & & & & -1 & & & & & & & & \\
 & & -1 & & & & & -1 & 3 & -1 & & & & & & & & & & & & \\
 & & & -1 & & & & & -1 & 4 & -1 & & & & -1 & & & & & & & \\
 & & & & -1 & & & & & -1 & 4 & -1 & & & & -1 & & & & & & \\
 & & & & & -1 & & & & & -1 & 4 & & & & & -1 & & & & & \\
 & & & & & & -1 & & & & & & 3 & -1 & & & & -1 & & & & \\
 & & & & & & & -1 & & & & & -1 & 3 & & & & & -1 & & & \\
 & & & & & & & & & -1 & & & & & 3 & -1 & & & & -1 & & \\
 & & & & & & & & & & -1 & & & & -1 & 4 & -1 & & & & -1 & \\
 & & & & & & & & & & & -1 & & & & -1 & 4 & & & & & -1 \\
 & & & & & & & & & & & & -1 & & & & & 3 & -1 & & & \\
 & & & & & & & & & & & & & -1 & & & & -1 & 3 & & & \\
 & & & & & & & & & & & & & & -1 & & & & & 3 & -1 & \\
 & & & & & & & & & & & & & & & -1 & & & & -1 & 4 & -1 \\
 & & & & & & & & & & & & & & & & -1 & & & & -1 & 4
\end{pmatrix}
\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \\ v_{11} \\ v_{12} \\ v_{13} \\ v_{14} \\ v_{15} \\ v_{16} \\ v_{17} \\ v_{18} \\ v_{19} \\ v_{20} \\ v_{21} \\ v_{22} \end{pmatrix}
=
\begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 5 \\ 5 \\ 25 \\ 25 \\ 10 \end{pmatrix}$$

Substituting these into the grid, we have

$$\begin{pmatrix}
5 & 5 & 5 & 5 & 5 & 5 & 5 & 5 \\
* & 5.33 & 5.50 & 5.88 & 6.37 & 6.27 & 5.72 & 5 \\
* & 5.50 & 5.78 & 6.66 & 8.32 & 8.00 & 6.60 & 5 \\
* & 5.39 & 5.47 & * & 12.23 & 10.83 & 7.68 & 5 \\
* & 5.20 & 5.22 & * & 17.55 & 15.41 & 8.27 & 5 \\
5 & 5 & 5 & 5 & 25 & 25 & 5 & 5
\end{pmatrix}$$